Assignment 6

1. Show that f is continuous from (X, d) to (Y, ρ) if and only if $f^{-1}(F)$ is closed in X whenever F is closed in Y.

2. Identify the boundary points, interior points, interior and closure of the following sets in $\mathbb{R}$:

   (a) $[1, 2) \cup (2, 5) \cup \{10\}$.

   (b) $[0, 1] \cap \mathbb{Q}$.

   (c) $\bigcup_{k=1}^{\infty}(1/(k+1), 1/k)$.

   (d) $\{1, 2, 3, \cdots\}$.

3. Identify the boundary points, interior points, interior and closure of the following sets in $\mathbb{R}^2$:

   (a) $R \equiv [0, 1) \times [2, 3) \cup \{0\} \times (3, 5)$.

   (b) $\{(x, y) : 1 < x^2 + y^2 \leq 9\}$.

   (c) $\mathbb{R}^2 \setminus \{(1, 0), (1/2, 0), (1/3, 0), (1/4, 0), \cdots\}$.

4. Describe the closure and interior of the following sets in $C[0, 1]$:

   (a) $\{f : \ f(x) > -1, \ \forall x \in [0, 1]\}$.

   (b) $\{f : \ f(0) = f(1)\}$.

5. Let A and B be subsets of (X, d). Show that $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

6. Show that $\overline{E} = \{x \in X : \ d(x, E) = 0\}$ for every non-empty $E \subset X$.

7. Show that f is continuous from (X, d) to (Y, ρ) if and only if for every $E \subset X$, $f(\overline{E}) \subset \overline{f(E)}$.